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# Application of Linear Programming Problem on Niger Mills Company PLC Calabar

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***Abstract* – This paper is on the application of Linear Programming Problem on Niger Mills Company PLC Calabar, Cross River State, Nigeria and intended to determine the quantity of Golden Penny flour (50kg bags), Golden Penny semovita (10kg bags) and Wheat offals (50kg bags) that Niger Mills Company PLC Calabar should produce in a day in order to maximize profit, given the constraints posed in the production process. A problem of this nature was identified as a linear programming problem, formulated in mathematical terms and solved by the available computer soft wares (Excel Solver and TORA) for solving Linear Programming problems. The solution obtained reveals that 576 (50kg) bags of flour, non presently for Semovita and wheat offals should be produced daily, in order for the company to achieve a maximum daily profit of ₦161, 280.00. Sensitivity analysis was also conducted to review the following; changing the objective function coefficient for a variable, forcing a variable which is currently zero to be non-zero, and changing the right-hand side of a constraint. The sensitivity analysis revealed that for Semovita (variable $X_2$), the objective function needs to change by 60.00000091 (increase since we are maximizing) before it becomes non-zero. In order words, referring back to our original situation in this research work, the profit per unit on semovita would need to increase by 60.00000091 before it would be profitable to produce any of semovita products. Similarly, the profit per unit on W/offals would need to increase by 45.0000002 before it would be profitable to produce any of W/offals products.**

***Keywords* – Linear Programming Problem, Optimization Problem, Mathematical Programming, Sensitivity Analysis, Simplex Method.**

## INTRODUCTION

Wheat flour consumption in Nigeria has been growing by leaps and bounds, but this does not necessarily connote readiness for wheat millers. The government, in its desire to help the nation's farmers, had imposed a requirement (which was to have started in July 2006) that millers blend 10% cassava flour into all their wheat flour production. However, compliance was an uphill task for millers, given the difficulties of proceeding, storing and transporting sufficient quantities of good quality cassava flour, not to mention the negative impact on bread quality and other products made from such a blend, considering the choosy propensities of Nigeria consumers. The government's objective wasn't successful in the end. Up until the second half of 2008, the industry was negatively impacted by the global food crises that spiked up the prices of international agricultural commodities for example, wheat.

Forecast for certain agricultural commodities made available recently from the world's apex organization overseeing matters of the stomach, the United Nations' Food and Agriculture Organization (FAO), will generally be subject of concern to many consumers and industrialists, locally and internationally, during the year. Events that will shape their supply and price have started since about the second half of 2013 and are likely to sustain their influences as this 2014 unfolds. Experts' views from FAO, dealing with specific areas of produce, noted that the world cereal production is expected to experience an increase and prices may come down to some extent as high prices earlier last year boosted plantings and weather conditions were generally favourable. This prospect has already led to a sharp drop in international prices of most cereals from their peaks during the first half of 2013.

From the expectations raised in the forecast, food and agriculture situation in Nigeria this year may be a strong reflection of the situation elsewhere as events all over the world may have strong influences on local situation. In Nigeria, grains serve two main purposes: for human consumption and for animal feed production. The animal feed aspect is known for poultry and fisheries. Events are thus expected to have significant effect on these industries this year. Separate from the grain cost aspect of the Milling business is the cost of power generation, infrastructure and even distribution.

## STATEMENT OF THE PROBLEM

There is need to explore how Niger Mills Company Calabar can achieve maximum profit from the production and sales of Golden Penny Flour, Golden Penny Semovita, and Wheat Offals in view of constraints inherent in the production process. To my widest imagination, no statistical studies have been conducted previously to determine the quantity of each product that Niger Mills Company Calabar should produce in order to maximize profit, therefore it becomes a major problem that the researcher used the company as a case study.

Another problem is a tendency for the many component of the organization to grow into relatively autonomous empires with their own goals and value systems, thereby losing sight of how their activities and objectives mesh with those of the overall organization. What is best for one component frequently is detrimental to another, so they may end up working at cross-purposes. A related problem is that as the complexity and specialization in an organization increase, it becomes more and more difficult to allocate its available resources to its various activities in a way that is most effective for the organization as whole. These kinds of problems and the need to find a better way to resolve them provided the environment for the emergence of operation research.

## Related Literature Review

Much work have been done, which in one way or the other relates to this work. Hence, it becomes necessary to review other people's work.

Nonso (2005) in his work on application of Linear programming for managerial decision found out how an organization can have effective control over materials for input during production. He observed that units produced must be assumed as what is sold in order to achieve the company's goal.

Ezema and Amakom (2012) worked on the optimizing profit with the linear programming model: A focus on Golden plastic industry limited, Enugu, 2012. The result they had showed that only 2 sizes of the total 8 'PVC' pipes should be produced.

Khan et al. (2011) in their work, optimal production levels for the different product manufactured at ICL, a multinational Company in Pakistan has a result that showed that the amount was raised by changing production patterns within the first, second, third and fourth digit respectively. The intention of this paper is therefore to determine the optimum production capacity of Usmer water company Uyo, Nigeria.

Adamu (2013) carried out a research on application of parametric linear programming in coca-cola company using a developed algorithm. In his paper, the critically examined parametric linear programming problem with interval in the coefficients of the objective function and the composition of coca-cola, Fanta, Sprite and Schweppes soft drink. The findings of soft drinks production in coca-cola company in Kaduna, Nigeria were studied and the problem formulated. The formulated problem was tested using the developed computer program by varying the parameter at regular interval and obtaining the corresponding values of the objective function. The final result of the research showed that as the parameter values were increased resulted in the increased of the values of the objective function and the developed computer program generate these results faster, and accurate then the other methods.

Fagoyinbo, et al. (2013) carried out a research on maximization of profit in manufacturing industries using linear programming techniques in GEEPEE Nigeria limited. The research work applied the concept of revised simplex method; an aspect of linear programming to solving industrial problem with the aim of maximizing profit. The industry GEEPEE Nigeria Limited specializes in production of tanks of various types. Four different types of tank were sampled for the study, which are the Combo, Atlas, Rambo and Jumbo tanks of various sizes. Based on the analysis of the data collected, it was observed that, given the amount of materials available, polyethylene (Rubber) and Oxy-acetylene (Gas) used in the production of the different sizes of the product, Combo tanks assumed more objective value contribution and gave maximum profit at a given level of production capacity.

Anieting et al. (2013) worked on application of linear programming technique in the determination of optimum capacity. In their study, they dealt with applying Linear Programming Technique in the determination of optimum linear programming technique to determine optimum production of Usmer Water Company, Uyo, Akwa-Ibom State, Nigeira. TORA software was used in the analysis of the data using big M-method. The result showed the values of the decision variables $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$ to be 95, 0, 5.9, 10 and 17 respectively. Sensitivity analysis of the problem was also discussed.

## Formulating a Linear Programming Model

### *Basic Concepts*

Suppose that the system under study (which may be one actually in existence or one which we wish to design) is a complex of machines, people, facilities, and supplies. It has certain over-all reason for its existence. For the military it may be to provide striking force, or for industry it may be to produce certain types of products.

The linear programming approach is to consider a system as decomposable into a number of elementary functions, the activities. An activity is thought of as a kind of "black box" into which flow tangible inputs, such as men, material, and equipment, and out of which may flow the products of manufacture, or the trained crews of the military. What happens to the inputs inside the "box" is the concern of the engineer or the educator; to the programmer, only the rates of flow into and out of the activity are of interest. The various kinds of flow are called items. The quantity of each activity is called the activity level. To change the activity level it is necessary to change the flows into and out of the activity.

### *Optimization Problem*

Optimization problems are problems that seek to maximize or minimize a given quantity called the objective function which depends on a finite number of input variables. These input variables may be independent or related through one or more constraints (Inyama; 2006).

### *Programming Problems*

A programming problem is a class of problems that determines the optimal allocation of limited resources to meet given objectives. The resources may be men, materials, machine and land. However, it generally aims at minimizing cost and maximizing profit.

### *Mathematical Programming*

If the objective and constraints of an optimization problem are given as mathematical functions and functional relations, it is called mathematical programming. It is generally given as:

$$\left.\begin{array}{ll}\text{Max. or Min.} & f = f(x) \\ \text{Subject to:} & g_1(\underline{x}) \leq \text{or} \geq b_i\end{array}\right\} \quad (1)$$

where $(\underline{X}) = (x_1, x_2, \ldots, x_n)$, $i = 1, 2, \ldots, m$

If $g_i = 0$ and $b_i = 0$ $\forall_1$; $i = 1, 2, \ldots, m$ then Equation(1) is called an unconstrained mathematical programming problem.

### *Linear Programming Problem*

This is a type of mathematical programming problem in which the objective function $f(\underline{x})$, and constraint, $g_i(\underline{x})$; $i=1,2,..,m$ are all linear functions. Let us consider the following general linear programming problem:

$$\left.\begin{array}{ll}\text{Min. or Max. } Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n & \\ \text{Subject to:} \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq = \geq b_1 & \\ \quad a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq = \geq b_2 & \\ \quad \vdots \qquad \vdots & \\ \quad a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq = \geq b_m & \\ \quad x_1, x_2, \ldots, x_n \geq 0 & \end{array}\right\} \quad (2)$$

This is a problem in n variables with m constraints. It could be a production planning problem with n products and m scarce resources.

The inequalities of Equation (1) may be converted into standard form by introduction of nonnegative S-variables (slack variables), which stands for unused capacities in the production planning case. This form is called the canonical form. Hence, instead of Equation (1), we may convert it into standard form according to Inyama (2007) as;

$$\left.\begin{array}{l}\text{Min. or Max. } Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n \\ \text{Subject to:} \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + S_1 = b_1 \\ \quad a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + S_2 = b_2 \\ \quad \vdots \qquad \vdots \\ \quad a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + S_m = b_m \\ \quad x_1, x_2, \ldots, x_n, S_1, S_2, \ldots, S_n \geq 0\end{array}\right\} \quad (3)$$

Equation (3) can be written as

$$\left.\begin{array}{ll}\text{Max. or Min.} & Z = C^T X \\ \text{Subject to:} & AX = \underline{b}\end{array}\right\} \quad (4)$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ldots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

### *Characteristics of Linear Programming Problems*

The following are the main characteristics of linear programming problems.

1) The decision variables ($x_j$, j, 1… n) in the problem are not negative. That is, they are either zero or positive.
2) The criterion for selecting the best values of the decision variable must be expressed as a linear function of these variables without any cross product. This is called the objective function.
3) The operating rules are restrictions governing the process can be expressed as a set of linear equations or inequalities. This is called the constraint set.

### *Transformation of Real Life Problems into LPP*

To formulate a linear programming model for a real life problem, we must adopt the following four steps.

1) Define the input variables say $x_j$, $j = 1, 2, \ldots, n$.
2) Determine the quantity to be maximized or minimized (optimized) and express it as a linear mathematical function. This constitutes the objective function.
3) Identify all stated requirements restrictions and limitations and express them as linear mathematical functions. These constitute the constraints.
4) Identify all other hidden conditions that are not clearly stated in the real-life problem but are obvious from the real life situation that is being modeled. Express these also as mathematical functions. These will also be a part of the constraints.

## Simplex Method

The simplex method which was developed by G.B. Dantzig is a systematic procedure for solving a linear programming problem by moving from one extreme point to another extreme point with a better objective value. This process continues until an optimal extreme point is reached or else until extreme direction with $C^Td_j < 0$ is found. In the later ease, we conclude that the optimal objective value is unbounded.

### *Tableau Format of the Simplex Method*

The linear programming problem (LPP) in standard form is given as

Max. $Z = C^TX$

Subject to: $AX = \underline{b}$ .

$C^T$ can be decomposed into two parts: basic, $C_B^T$ and non-basic, $C_N^T$, that is

$$C^T = C_B^T + C_N^T$$

In the same way A and X can be decomposed into

$A = [B, N]$

$X = [X_B, X_N]$

Hence the problem becomes

$$\text{Max. } Z = C_B^T X_B + C_N^T X_n$$

$$\text{Subject to: } BX_B + NX_N = \underline{b}$$

This is achieved by choosing the original variables ($x_1$, $x_2$, …, $x_n$) as the non-basic variables and slack variables ($S_1$, $S_2$, $S_3$, …, $S_n$) as the basic variables. Hence the tableau format of the simplex method for a maximization problem is Table 1.

Table 1: Tableau Format for a Maximization Problem in Simplex Method

| Basic | Z | $X_N^T$ | $X_B^T$ | RHS |
|---|---|---|---|---|
| Z | ① | $C_N^T$ | $C_B^T$ | $\underline{O}$ |
| $X_B$ | $\underline{O}$ | N | B | $\underline{b}$ |

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*Minimization Problem*

To solve a minimization problem we convert it to an equivalent maximization problem by multiplying the objective function of the minimization problem by -1, and then use the simplex method as outlined for a maximization problem. Hence the problem

Min. $Z = C^T X$

Subject to: $AX = \underline{b}$

becomes

Max. $Z' = -C^T X = -C_N^T X_N - C_B^T X_B$

Subject to: $AX = NX_N + BX_B = \underline{b}$

Hence the tableau becomes

Table 2: Tableau Format for a Minimization Problem in Simplex Method

| Basic | Z′ | $X_N^T$ | $X_B^T$ | RHS |
|---|---|---|---|---|
| Z | ① | $-C_N^T$ | $-C_B^T$ | $\underline{O}$ |
| $X_B$ | $\underline{O}$ | N | B | $\underline{b}$ |

*Optimality Condition for the Simplex Method*

In a maximization problem, if all the non-basic variables have non-positive coefficients in the z (object function) equation of the current solution, then the current solution is optimal.

*Summary of the Simplex Method*

The basic steps of the simplex method for a maximization problem are as follows:

*Step 1:* Put the problem in the standard form.

*Step 2:* Using the standard form determine the initial basic feasible solution.

*Step 3:* Check if the current feasible solution is optimal. If the current feasible solution is non-positive (for a maximization problem), stop, the current solution is optimal, otherwise go to step 4.

*Step 4:* Select a non-basic variable to enter the basis, (to become the new basic variable). The general rule is to select the non-basic variable with the largest positive coefficient in the objective row (for a maximization problem). Break tie arbitrarily.

*Step 5:* Determine (from the current basic variables) the basic variable that will leave the basis (that is, the one that will be replaced by the non-basic variables selected in step 4). The leaving variable is the basic variable having the smallest ratio of the RHS of the constraints equation to be associated with positive coefficient of the entering variable. Break tie arbitrarily.

*Note:* The coefficient of the entering variable in the constraints equation is called the pivot element. The equation involving the leaving variable is called the pivot equation.

*Step 6:* After determining the entering and leaving variables, obtain the new pivot equation. New pivot equation = (old pivot equation)/(pivot element).

*Step 7:* Using pivot operation (Gaussian Jordan elimination method) eliminate the entering variable from all other equations including the z-equation, go back to step 3.

## SENSITIVITY ANALYSIS

Sensitivity analysis is concerned with how changes in an LP's parameters affect the optimal solution. After the optimal solution of a linear programming problem has been computed for a given set of data, it frequently happens that these data change to some extend; it may also be that the data are not known with certainty. It is, of course, always possible to find the optimal solution for a slightly different set of data by solving the problem again, but a more fruitful approach is to ask for which changes in the data the present optimal solution remains optimal. This is called Sensitivity Analysis. Changes of all data may be considered, but here we shall only consider variations in the coefficient of the objective function and changes in the right-hand side of the constraints. The sensitivity of a solution to changes in the data gives us insight into possible technological improvements in the process being modeled. For instance, it might be that the available resources are not balanced properly and the primary issue is not to resolve the most effective allocation of these resources, but to investigate what additional resources should be acquired to eliminate possible bottlenecks. Sensitivity analysis provides an invaluable tool for addressing such issues.

Solving a linear program usually provides more information about an optimal solution than merely the values of the decision variables. Associated with an optimal solution is a shadow price (also referred to as dual variables) for the constraints.

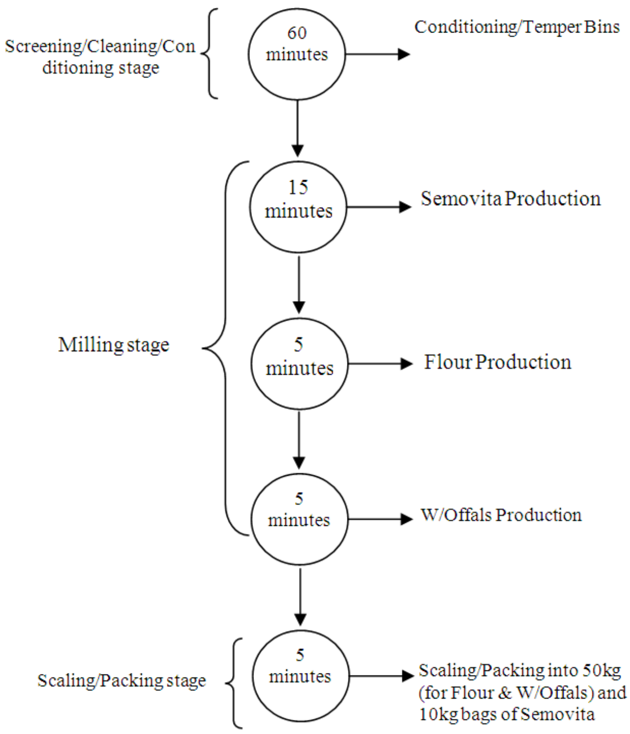


Fig.1. Sequence of Production Process

Niger Mills Company produces three different products namely, Flour, Semovita and Wheat Offals (Animals Feed) from one source of Raw Material – Wheat. These three products (Flour, Semovita and Wheat Offals) are produced by passing sequentially through three different stages of production viz; the Cleaning/Screening/ Conditioning stage, the milling stage and the scaling/packing stage. The diagram above shows the sequence of this production process.

From the above, 250 tons of Wheat per day is poured into the condition/temper bins to be cleaned and conditioned before Milling. It usually takes 60 minutes for the wheat to be cleaned and ready for milling. When ready for milling, the same quantity is passed into the milling stage for grinding. It takes 15 minutes for Semovita to be produced from the ground wheat. Further grinding of coarse and find middling leads to the production of Flour in an additional 5 minutes. It takes another 5 minutes to produce Wheat offals.

Finally, the last stage of production (Scaling/Packing) also takes another 5 minutes to scale and packed Flour and Wheat Offals into their respective 50kg bags and 3 minutes to scaled and packed Semovita into its 10kg bag.

## Description of Bound in the Market Constraint

From the sales department of the company, the maximum quantity of each product (i.e. maximum monthly demand) that is sold in a month is given as

Flour = 180,060 (50kg) bags

Semovita = 43,280 (10kg) bags and

Wheat Offals = 29,382 (50kg) bags

Therefore, these quantities 180,060 (50kg) bags of flour, 43,280 (10kg) bags of Semovita and 29,382 (50kg) bags of wheat offals are the respective bounds of market constraints and this is shown clearly in Table 9 (see Appendix ).

### *Estimation of Bound in the Time Constraint*

To estimate the bound in time constraint, we recall that three are three different stages in the production process namely, screening/cleaning/conditioning stage, milling stage and the scaling/packing stage.

We were also told in this paper that it took 60 minutes to screen, clean and condition that wheat before milling. At the milling stage, once the process is on, it normally takes 15 minutes for the first product (semovita) to be produced, 20 minutes for flour to be produced and 25 minutes for wheat to be produced. Finally, in the scaling/packing stage, it takes 5 minutes for flour and wheat offals to be scaled and packed into their respective 50kg bags, and only 3 minutes for semovita to be scaled and packed into 10kg bag. The stages time allocated to the month, therefore, to estimate the limit was multiply 24hrs x 60 minutes x 24 days = 34,560 minutes.

Therefore, 34,560 minutes is the bound for time constraint, which means that the company can only run for 34, 560 minutes. This is also shown clearly in Table 7 (see Appendix)

### *Description of Bound in the Raw Material Constraint*

It was also observed in this research, that 6000tons of raw material (wheat) is imported monthly by the company for the production of flour, semovita and wheat offals. This means that, 6000tons of wheat is the minimum monthly requirement by the company and hence, it is the bound for raw material constraint.

## Problem Formulation

To formulate the mathematical model for this problem, let $X_1$ be the quantity to be produced of flour, $X_2$ be the quantity to be produced of semovita and $X_3$ be the quantity to be produced of wheat offal respectively, and $f$ is the total profit that would be made from the sales of these three items. Thus, $X_1$, $X_2$ and $X_3$ are the decision variables for the model. The objective is to choose the values of $X_1$, $X_2$ and $X_3$ so as to maximize the profit function.

$$f = C_1X_1 + C_2X_2 + C_3X_3 \qquad (5)$$

subject to the restrictions imposed on their values by the limited monthly availability.

From the accounts department, it was learnt that the cost of producing a 50kg bag of flour is ₦5,670.00, while that of producing a 10kg bag of semovita is ₦1,130.00 and that of producing a 50kg bag of wheat offals is put at ₦1,315.00 as shown in Table 8 (see Appendix). Also from the sales department, it was learnt that the selling price for a bag of flour is ₦5, 950.00, while the selling price for a bag of semovita is ₦1, 350.00 and that of a bag of wheat offal is ₦1, 550.00. By subtracting the direct cost of each product from the selling price, we have the net revenue (profit) of ₦280, ₦150 and ₦170 made on selling a bag of Flour, Semovita and Wheat offals.

Therefore, our profit per unit ($C_1$) of $X_1$ is ₦280.00, while per unit ($C_2$) of $X_2$ is ₦220 and profit per unit ($C_3$) of $X_3$ is ₦235.00 (as shown in Appendix C). Substituting these values in (5) yields

$$f = 2800X_1 + 220X_2 + 235X_3 \qquad (6)$$

This is the profit function that we seek to maximize.

The first row in Table 5 (see Appendix) implies that each unit of Flour produced would use $\frac{1}{20}$ of wheat (raw material), because one ton of wheat consumed, produces 200 (50kg) bags of flour, and a ton (= 1000kg), while each unit of Semovita produced would use $\frac{1}{100}$ of wheat (raw material), because one ton of wheat consumed, produces 100 (10kg) bags of Semovita and finally, each unit of Wheat offals produced would also use $\frac{1}{20}$ of the wheat (raw material), whereas only 6000tons is available in a month.

This restriction is expressed mathematically by the inequality:

$$\frac{1}{20X_1} + \frac{1}{100X_2} + \frac{1}{20X_3} \leq 6000 \qquad (7)$$

Similarly, it took 60 minutes to clean, screen and condition the wheat before milling; we note that this screening, cleaning and conditioning process is done by

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machines, and these machines runs for a maximum of 34,560 minutes per month. This also imposes the restriction that:

$60X_1 + 60X_2 + 60X_3 \leq 34,560$ (8)

At the milling stage, it took 15minutes to produces semovita, 20minutes to produce flour and 25 minutes to produce wheat offals. This restriction is expressed mathematically by the inequality:

$20X_1 + 15X_2 + 25X_3 \leq 34,560$ (9)

The final stage of production process is the scaling/packing stage. This takes 5minutes to scaled and packed four and wheat offals into their respective bags of 50kg and 3minutes to scaled and packed semovita into its 10kg, and leads to the restriction that:

$5X_1 + 3X_2 + 5X_2 \leq 34,560$ (10)

Finally, the maximum monthly demand (i.e. the exact quantity of each product that is produced and sold completely) for each product – Flour, Semovita and Wheat Offals is 180,060 (50kg) bags, 43,280 (10kg) bags and 29,382 (50kg) bags. Therefore, the mathematical statement of these demands for Flour, Semovita and Wheat Offals is:

$X_1 \leq 180,060$ (11)

$X_2 \leq 43,280$ (12)

$X_3 \leq 29,382$ (13)

Furthermore, in any feasible production program, the quantities produced cannot be negative; hence we have the following three inequalities

$X_1 \geq 0,\ X_2 \geq 0,\ X_3 \geq 0$ (14)

The problem can now be formulated in mathematical terms as that of finding values of $X_1$, $X_2$ and $X_3$ which satisfy the inequalities (7), (8), (9), (10), (11), (12), (13) and (14) and for which f is given by (6) is maximized i.e. we seek to find values of $X_1$, $X_2$ and $X_3$ so as

Maximize $f = 280X_1 + 220X_2 + 235X_3$ (15)

Subject to

$$\left.\begin{aligned} &\frac{1}{20X_1} + \frac{1}{100X_2} + \frac{1}{20X_3} \leq 6000 \\ &60X_1 + 60X_2 + 60X_3 \leq 34560 \\ &20X_1 + 15X_2 + 25X_3 \leq 34560 \\ &5X_1 + 3X_2 + 5X_3 \leq 34560 \\ &X_1 \leq 180{,}060\ X_2 \leq 43,\ 280\ X_3 \leq 29382 \end{aligned}\right\} \quad (16)$$

$X_j \geq 0$, j = 1, 2, 3 (17)

This problem has all the characteristics of a linear programming problem, in a general linear programming problem; a linear function [such as (15)] is maximized subject to a number of linear inequalities and equations [such as (16) and (17)]. Therefore, it is a linear programming problem.

## Linear Programming Solution by Excel Solver

The problem data are inputted into the system as shown in Table 3:

Table 3: Excel Solver Solution for the Linear Programming Problem

| Niger Mills Company PLC Calabar | | | | | | |
|---|---|---|---|---|---|---|
| Input data | | | | | | |
| | $X_1$ | $X_2$ | $X_3$ | | | |
| | Flour | Semovita | W/offals | Total | | Limits |
| Objectives | 280 | 220 | 235 | 161280.00 | | |
| Raw materials | $\frac{1}{20}$ | $\frac{1}{100}$ | $\frac{1}{20}$ | 28.800 | <= | 6000 |
| Time 1 | 60 | 60 | 60 | 34560.00 | <= | 34560 |
| Time 2 | 20 | 15 | 25 | 11520.00 | <= | 34560 |
| | 5 | 3 | 5 | 2880.00 | <= | 34560 |
| Demand 1 | 1 | 0 | 0 | 576.00 | <= | 180060 |
| Demand 2 | 0 | 1 | 0 | 0.00 | <= | 43280 |
| Demand 3 | 0 | 0 | 1 | 0.00 | <= | 29382 |
| | >=0 | >=0 | >=0 | | | |
| Output results | | | | | | |
| | $X_1$ | $X_2$ | $X_3$ | f | | |
| Solution | 576.00 | 0 | 0 | 161280.00 | | |

Table 4: Excel Solver's Output Summary

| Target Cell (Max) | | | |
|---|---|---|---|
| **Cell** | **Name** | **Original Value** | **Final Value** |
| $E$3 | Objectives Total | 161280 | 161280 |
| Adjustable Cells | | | |
| **Cell** | **Name** | **Original Value** | **Final Value** |
| $B$15 | Solution Flour | 576 | 576 |
| $C$15 | Solution Semovita | 0 | 0 |
| $D$15 | Solution W/Offals | 0 | 0 |

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Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $E$4 | Raw Materials Total | 28.8 | $E$4<=$G$4 | Not Binding | 5971.2 |
| $E$5 | Time 1 Total | 34560 | $E$5<=$G$5 | Binding | 0 |
| $E$6 | Time 2 Total | 11520 | $E$6<=$G$6 | Not Binding | 23040 |
| $E$7 | Time 3 Total | 2880 | $E$7<=$G$7 | Not Binding | 31680 |
| $E$8 | Demand 1 Total | 576 | $E$8<=$G$8 | Not Binding | 179484 |
| $E$9 | Demand 2 Total | 0 | $E$9<=$G$9 | Not Binding | 43280 |
| $E$10 | Demand 3 Total | 0 | $E$10<=$G$10 | Not Binding | 29382 |
| $B$15 | Solution Flour | 576 | $B$15>=0 | Not Binding | 576 |
| $C$15 | Solution Semovita | 0 | $C$15>=0 | Binding | 0 |
| $D$15 | Solution W/Offals | 0 | $D$15>=0 | Binding | 0 |

*Sensitivity Analysis Output*

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$15 | Solution Flour | 576 | 0 | 280 | 1E+30 | 45.00000024 |
| $C$15 | Solution Semovita | 0 | -60.00000091 | 219.9999988 | 60.00000091 | 1E+30 |
| $D$15 | Solution W/Offals | 0 | -45.0000002 | 234.9999995 | 45.0000002 | 1E+30 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $E$4 | Raw Materials Total | 28.8 | 0 | 6000 | 1E+30 | 5971.2 |
| $E$5 | Time 1 Total | 34560 | 4.666666667 | 34560 | 69120 | 34560 |
| $E$6 | Time 2 Total | 11520 | 0 | 34560 | 1E+30 | 23040 |
| $E$7 | Time 3 Total | 2880 | 0 | 34560 | 1E+30 | 31680 |
| $E$8 | Demand 1 Total | 576 | 0 | 180060 | 1E+30 | 179484 |
| $E$9 | Demand 2 Total | 0 | 0 | 43280 | 1E+30 | 43280 |
| $E$10 | Demand 3 Total | 0 | 0 | 29382 | 1E+30 | 29382 |

The output of sensitivity analysis provides us with information relating to:

- changing the objective function coefficient for a variable
- forcing a variable which is currently zero to be non-zero
- changing the right-hand side of a constraint

*Changing the Objective Function Coefficient for a Variable*

In this case, we shall vary the coefficient of $X_1$ in the objective function. Currently, $X_1$ =576, $X_2$ =0 and $X_3$ = 0. The current solution value for $X_1$ is 576 and the current objective function coefficient for $X_1$ is 280. The Allowable Increase/Decrease columns tell us that, provided the coefficient of $X_1$ in the objective function lies between 280 + (1E+30) = and 280 – 45.00000024 = (to four decimal places), the values of the variables in the optimal LP solution will remain unchanged. In terms of the original problem, we are effectively saying that the decision to produce 576 of flour remains optimal even if the profit per unit on flour is not actually 280 (but lies in the range). Similar conclusions can be drawn about $X_1$ and $X_3$.

*Forcing a Variable Which Is Currently Zero to Be Non-Zero*

For the variables, the Reduced Cost column gives us, for each variable which is currently zero ($X_2$ and $X_3$), an estimate of how much the objective function will change if we make (force) that variable to be non-zero. It should be noted here that the value in the Reduced Cost column for a variable is often called the “opportunity cost” for the variable.

Hence, we have the table

| Variable | $X_2$ | $X_3$ |
|---|---|---|
| Reduced Cost (Opportunity Cost) | 60.00000091 | 45.0000002 |
| New value (= or >=) | $X_2$=A or $X_2$>=A | $X_3$=B $X_3$>=B |

Estimated objective function change 60.00000091A 45.0000002B where we ignore the sign of the reduced cost when constructing the above table. The objective function will always get worse (go down since we have a maximization problem) by at least this estimate. It should be noted that an alternative (and equally valid) interpretation of the reduced cost is the amount by which the objective function coefficient for a variable needs to change before that variable will become non-zero.

Hence, for Semovita (variable $X_2$), the objective function needs to change by 60.00000091 (increase since we are maximizing) before that it becomes non-zero. In order words, referring back to our original situation in this research work, the profit per unit on semovita would need to increase by 60.00000091 before it would be profitable to produce any of semovita products. Similarly, the profit per unit on W/offals would need to increase by 45.0000002 before it would be profitable to produce any of W/offals products.

*Changing the Right-Hand Side of a Constraint*

For each constraint the headed Shadow Price tells us exactly how much the objective function will change if we change the right-hand side of the corresponding constraint within the limits given in the Allowable Increase/Decrease columns.

Table 5: Allowable Increase/Decrease Columns for the Constraints

| Constraint | Raw materials | Time 1 | Time 2 | Time 3 | Demand 1 | Demand 2 | Demand 3 |
|---|---|---|---|---|---|---|---|
| Opportunity (Reduced) Cost (ignore sign) | 0 | 4.666666667 | 0 | 0 | 0 | 0 | 0 |
| Change in right-hand side | a | b | c | d | e | F | G |
| Objective function change | 0 | 4.666666667b | 0 | 0 | 0 | 0 | 0 |
| Lower limit for RHS | 28.8 | 0 | 11520 | 2880 | 576 | 0 | 0 |
| Current value for RHS | 6000 | 35560 | 34560 | 34560 | 180060 | 43280 | 29382 |
| Upper limit for RHS | - | 103680 | - | - | - | - | - |

For instance, for the time 1 constraint provided the right-hand side of that constraint remains between 34560 + 69120 = 103680 and 34560 – 34560 = 0, the objective function change will be exactly 4.666666667(change in right-hand side from 34560).

## Conclusion

In this study, we were able to identify the problem confronting Niger Mills Company Calabar as a linear programming problem, formulate a mathematical model that represents the essence of the problem, identify the functional constraints of the problem as well as solved the problem using the available computer software (Excel Solver), and TORA Software for solving linear programming problems. From the solution obtained, we were also able to determine an optimal production program for Niger Mills Company Calabar. The Niger Mills Company Calabar model may be viewed as a model that roughly represents how successful operations research studies are conducted.

## Recommendations

From the solutions obtained in the proceeding chapter, it is hereby recommended that:

1) Niger Mills Company Calabar should produce 576 (50kg) bags of flour per day, and stop presently to produce semovita and wheat offals, because of their non-optimality, to enable the company maximize profit.
2) Finally, it is also recommended that a statistical unit be created in the company so that qualified statisticians can be engaged to direct the collection, compilation and subsequent storage of statistical data on the product produced/sold by the company. Because, sometimes important data are not kept, and this will affect the result of any statistical studies conducted on the data as well as invalidate the recommendations of such studies.

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Mr. Andrew Iheawara obtained his Ordinary National Diploma (OND) in statistics and mathematics from Yaba College of Technology, Yaba Lagos Nigeria in 1984 and thereafter, he got his Higher National Diploma (HND) from Institute of Management and Techology (IMT), Enugu State Nigeria in 1987 from the department of statistics. Mr. Andrew obtained his Postgraduate Diploma (PGD) in statistics and Masters Degree (M.Sc.) both in Imo State University, Owerri Nigeria in 2010 and 2013 respectively. Mr. Andrew also obtained a Post graduate diploma certificate in Education from National Teachers' Institute, in affiliation with the Usman Dan-fodio University, Kaduna-Kaduna State Nigeria. Presently, he is the director, planning. Research and Statistics in the Imo State Ministry of Petroleum and Environment Nigeria.

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## APPENDIX

*Niger Mills Company PLC Calabar, Cross River State*

Table 6: Raw Material (Wheat Flour) Used, Quantity Produced, Sold and Not Sold From November 2012 – October 2013

| Month | Wheat used (tons) | | | Quantity produced | | | Quantity sold | | | Quantity not sold | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | GPF | GPS | WO | GPF | GPS | WO | GPF | GPS | WO | GPF | GPS | WO |
| Nov | 5564 | 446 | 1310 | 111280 | 44600 | 26200 | 111170 | 4430 | 26100 | 110 | 170 | 100 |
| Dec | 5508 | 442 | 1370 | 110160 | 44200 | 27400 | 110100 | 44000 | 27300 | 60 | 200 | 100 |
| Jan | 5536 | 444 | 1340 | 110720 | 44400 | 26800 | 110490 | 44210 | 26609 | 230 | 190 | 191 |
| Feb | 5480 | 440 | 1400 | 109600 | 44000 | 28000 | 109480 | 43874 | 27880 | 120 | 126 | 120 |
| March | 5452 | 438 | 1430 | 109040 | 43800 | 28600 | 109000 | 43610 | 28510 | 40 | 190 | 90 |
| Apr | 5425 | 435 | 1460 | 108500 | 43500 | 29200 | 108350 | 43320 | 29120 | 150 | 180 | 80 |
| May | 5397 | 433 | 1490 | 107940 | 43300 | 29800 | 107780 | 43120 | 29610 | 160 | 180 | 190 |
| June | 5369 | 431 | 1520 | 107380 | 43100 | 30400 | 107140 | 43000 | 30310 | 240 | 100 | 90 |
| July | 5285 | 425 | 1610 | 105700 | 42500 | 32200 | 105485 | 42312 | 32120 | 215 | 188 | 80 |
| Aug | 5313 | 427 | 1580 | 106260 | 42700 | 31600 | 106112 | 42600 | 31470 | 148 | 100 | 130 |
| Sep | 5257 | 423 | 1640 | 105140 | 42300 | 32800 | 105000 | 42150 | 32680 | 140 | 150 | 120 |
| Oct | 5341 | 429 | 1550 | 106820 | 42900 | 3100 | 106610 | 42730 | 30870 | 210 | 170 | 130 |
| Mean | 5410.58 | 434.42 | 1475.00 | 1082106 | 43441.67 | 29500 | 180059.75 | 43276.67 | 29381.58 | 151.92 | 162.00 | 118.42 |

Table 7: Production Time

| Production Stage | Golden Penny Flour | Golden Penny Semovita | Wheat Offals | Available Time Per Month |
|---|---|---|---|---|
| Screening/cleaning conditioning stage | 60 | 60 | 60 | 34560 |
| Milling stage | 20 | 15 | 25 | 34560 |
| Scaling/packing stage | 5 | 3 | 5 | 34560 |

Table 8: Direct Cost, Selling Price and Net Revenue Per Bag

| Item | Golden Penny Flour | Golden Penny Semovita | Wheat Offals |
|---|---|---|---|
| Direct cost (₦ per bag) | 5,670.00 | 1,130.00 | 1,315.00 |
| Selling price (₦ per bag) | 5,650.00 | 1,350.00 | 1,550.00 |
| Net revenue (₦ per bag) | 280.00 | 220.00 | 235.00 |

Table 9: Raw Material usage and Market (Demand)

| Item | Golden Penny Flour | Golden Penny Semovita | Wheat Offals | Monthly demand |
|---|---|---|---|---|
| Wheat flour (Raw material) | $\frac{1}{20}$ | $\frac{1}{100}$ | $\frac{1}{20}$ | |
| Demand for Golden penny flour | 1 | – | – | 180,060 (50kg) bags |
| Demand for Golden Penny Semovita | – | 1 | – | 43,280 (10kg) bags |
| Demand for Wheat offals | – | – | 1 | 29,382 (50kg) bags |

Table 10: Data for Calaber Flour Mills Limited

| Item | Golden Penny Flour | Golden Penny Semovita | Wheat Offals | Monthly Availability |
|---|---|---|---|---|
| Raw material (wheat flour in ton) | $\frac{1}{20}$ | $\frac{1}{100}$ | $\frac{1}{20}$ | 6000 |
| Conditioning stage | 60 | 60 | 60 | 34560 |
| Milling stage | 20 | 15 | 25 | 34560 |
| Scaling/packing stage | 5 | 3 | 5 | 34560 |
| Demand for Golden penny flour | 1 | – | – | 180,060 |
| Demand for Golden Penny Semovita | – | 1 | – | 43,280 |
| Demand for Wheat offals | – | – | 1 | 29382 |
| Net revenue per unit | 280 | 220 | 235 | |

Table 11: Fixed Costs

| Month | Personnel cost (₦) | Cost of Electricity (₦) | Utility cost (₦) |
|---|---|---|---|
| Nov | 8,415,215.00 | 7,760,000.00 | 4,500,000.00 |
| Dec | 8,515,412.00 | 7,770,000.00 | 4,400,000.00 |
| Jan | 8,416,216.00 | 7,780,000.00 | 4,600,000.00 |
| Feb | 8,475,414.00 | 7,660,000.00 | 4,500,000.00 |
| March | 8,316,312.00 | 7,750,000.00 | 4,600,000.00 |
| Apr | 8,295,415.00 | 7,860,000.00 | 4,700,000.00 |
| May | 8,425,276.00 | 7,650,000.00 | 4,500,000.00 |
| June | 8,435,295.00 | 7,760,000.00 | 4,600,000.00 |
| July | 8,475,318.00 | 7,745,000.00 | 4,500,000.00 |
| Aug | 8,325,215.00 | 7,780,000.00 | 4,600,000.00 |
| Sep | 8,313,245.00 | 7,770,000.00 | 4,500,000.00 |
| Oct | 8,475,318.00 | 7,650,000.00 | 4,650,000.00 |
| Mean | 8,406,970.92 | 7,744,583.33 | 4,554.166.67 |