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Mathematical Transformations using Geogebra: An Analysis of a Video Lesson

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Abstract **- The use of Information and Communication Technologies (ICTs) in teaching and learning of subjects such as mathematics has been encouraged the world over. Among the available mathematical ICT software, Geogebra was chosen for this study. Geogebra is a free dynamic mathematical software which incorporates geometry, algebra and calculus features in one programme. The study, guided by the model-centred learning theory as well as the TPACK and Mathematical Knowledge for Teaching frameworks, sought to analyse a video lesson on transformations of trigonometrical functions to reveal the content knowledge, attitudes and behaviours of the teacher and learners as they interacted with Geogebra. Using systematic sampling, five out of 35 Advanced Level learners were selected. A volunteer research assistant then took a video of the teacher while she was using Geogebra to teach and illustrate, to the five leaners, the relationship between trigonometrical functions, their graphs and transformations. The research assistant also took a video of one of the learners illustrating and demonstrating, using Geogebra, the transformations of the trigonometrical functions. The study was carried out during COVID-19 lockdown period, hence the small sample size. To analyse data, the researchers watched the videos at least five times, looked for the number of times certain terms and concepts were mentioned or explained as well as the importance, usefulness or value attached to Geogebra and to the video lesson. Some emerging themes were then categorised. Findings of this study could help to shape the way videos and ICTs are perceived in model-centred and online mathematics learning which have recently become the norm.**

Keywords **- mathematical transformations, trigonometrical functions, Geogebra, video lesson, video analysis.**

I. INTRODUCTION

The use of Information and Communication Technologies (ICTs) in teaching and learning of school subjects such as mathematics has been encouraged the world over. For instance, the National Council of Teachers of Mathematics (NCTM, 2000, p. 11)'s technology principle states that, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." In South Africa, Grades 10-12 leaners should be able to "use available technology (the minimum being a modern scientific calculator) in calculations and in the development of models." (Department of Education, 2003, p.10) whilst the Government of Zimbabwe has envisaged the increased usage of ICTs in the teaching and learning of primary and secondary school subjects (Government of Zimbabwe, 2016).

ICT is included in all the Form 5-6 pathways in the new Zimbabwe curriculum (Ministry of Primary and Secondary Education, 2017) and in particular the Form 5-6 Additional and Pure Math syllabi include ICT tools and 'talking books' and/or software as some of the suggested resources (Ministry of Primary and Secondary Education, 2015a, 2015c). The teaching and learning of Pure Mathematics must be learner centred and ICT driven (Ministry of Primary and Secondary Education, 2015b, p.2). In this regard, some of the Form 5-6 Pure Mathematics aims are as follows:

3.1. Acquire Enterprising Skills through Modelling Research and Project Based Learning.

3.3. Develop an appreciation of the applicability, creativity and power of pure mathematics in solving a broad range of problems

3.5. Appreciate the use of ICT tools in solving pure mathematical problems (2015c, p.2).

Also, some of the objectives state:

4.1. Make use of a variety of mathematical skills (including graphing, proving, modelling, finding pattern and problem solving) in the learning and application of Pure Mathematics

4.3. Produce imaginative and creative work arising from pure mathematical ideas

4.9. Conduct research projects including those related to enterprise. (2015c, p. 2).

Objective 4.9 is similar to Aim 3.7 for the Pure Math Form 3-4 syllabus (Ministry of Primary and Secondary Education, 2015b): "acquire enterprise skills in an indigenised and globalised economy through research and project-based learning." Thus, at least in theory, the mathematics syllabi for Zimbabwe secondary schools are rich in technology or ICT-related aims and objectives, the imparting of modelling, creativity and entrepreneurial skills, and it will remain to be established whether there is effective implementation of such.

Instead of the traditional chalk and talk approaches, instructional methodologies for ICT usage may include video-based demonstration, affective intervention, learning with children, and social and cognitive scaffoldings (Bu, Mumba, Henson & Wright, 2013; Savola, 2008). However, while it is acknowledged that, "…technology is giving educators an opportunity to open new doors to mathematical understanding for our learners" (Heid, 2002, p.662), most people are failing to take advantage of that opportunity.

Among the available mathematical ICT software, Geogebra is often chosen for research studies. Geogebra is a free dynamic mathematical software which incorporates geometry, algebra and calculus features in one programme. It was developed by Markus Hohenwarter as part of his masters' thesis (Hohenwarter, Jarvis & Lavicza, 2009). Geogebra can be freely accessed from the internet, easily installed in a computer and used offline (Mthethwa, Bayaga, Bosse & Williams, 2020). It can be used by teachers only (teacher-centred approach) or leaners only (leaner-centred approach) or by both during classroom lessons or even at home (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008) and is also suitable for any level from primary school to university.

II. Purpose of the Study

This study seeks to demonstrate how technology can be integrated in mathematics teaching and learning. In particular, the study seeks to analyse a video lesson on transformations of trigonometrical functions to reveal the content knowledge, attitudes and behaviours of teachers and learners as they interact with Geogebra.

III. Research Questions

1. What attitudes and behaviours do teachers and learners exhibit regarding the use of Geogebra in illustrating mathematical transformations of trigonometrical functions?
2. To what extent can Geogebra-based video lessons impact on the technological content knowledge of the learners?
3. How can the content and pedagogical knowledge of the teacher be analysed through Geogebra-based video

lesson?

IV. Review of Related Literature

Although the use of technology in the teaching and learning of mathematics has gained more prominence in recent years, Kondratieva (2013) says that dispute still exist on whether students will still be able to do some calculations without the aid of technological gadgets such as the calculator. Another cause for concern for both teachers and leaners could be how to choose the appropriate gadgets and related software (Lavicza & Koren, 2015; Kondratieva, 2013). Among the various existing software, Geogebra seems to be more appropriate since it enables students to move from static exploration to dynamic exploration (Ljajko & Ibro, 2013).

The characteristics of Geogebra and its uses are, according to Choi (2010), that:

- It enhances creativity and innovation skills in learners.
- It encourages collaborative learning.
- It motivates students and humanizes the classroom.
- It supports learner-centred education.

Geogebra also supports entrepreneurship among students in the sense that out of the mathematical and ICT demonstrations and constructions, artefacts can be designed and produced, and sold or used for entertainment. For example, students in Korea were taught to use Geogebra to produce models of the solar system, amusement park, and a fountain 'with quadratic functions.' (Choi, 2010). By using the dynamic mathematical software, the real world can be brought into the classroom (Pierce & Stacey, 2011). This is in line with Zimbabwe's Education 5.0 philosophy which advocates for teaching coupled with research and innovation, a philosophy which can achieve the objectives of education for economic transformation or entrepreneurship education (Mauchi, Karambakuwa, Gopo, Kosmas, Mangwende & Gombarume, 2011; Sofoluwe, Shokunbi, Raimi & Ajewole, 2013; https://library.buse.ac.zw/docs/gvt-publications/higher-edu-plan.pdf.

The teacher can employ learner centred instructional designs through using Geogebra based video lessons. Moreso, when the recent Covid-19 pandemic started, classroom teaching was replaced by distance teaching and learning through online digital technologies and self-study ways (Marinoni, van't Land & Jensen, 2020). In this regard, video demonstrations appear to be appropriate. In fact, the advantages of video lessons or demonstrations during teaching have been documented. According to Savola (2008), some of them are that:

1) The social, didactic, linguistic, and other types of patterns occurring in the classroom can be detected by playing the recording back at different speeds, or out of sequence.
2) A more truthful account of what has actually happened can be obtained with video rather than questionnaires or surveys, which are based on recollections or opinions.
3) Video is relatively bias-free.
4) Analysis and re-analysis of the video lesson is often unlimited.
5) Video analysis allows for multiple viewpoints.
6) Video analysis facilitates the integration of qualitative and quantitative methods.

7) Video footage can be reduced to still pictures, transcripts, and other forms of event portrayal.

8) Video recordings are multidimensional and contain lots of information.

Some research on the impact of Geogebra on students' mathematics achievement has been done. In South Africa, it was reported that Grade 11 pupils learning geometry with Geogebra did better at solving problems and verifying or justifying the answers than learners learning the same topic through the traditional approach (lecture based). It was also found that the use of Geogebra had positive effects on teachers' beliefs about teaching and learning (Mthethwa et al. 2020). In a different study in Malaysia, Saha, Ayub and Tarmizi (2010) conducted a study on the effects of Geogebra on mathematics achievement and concluded that students who used Geogebra to learn Coordinate Geometry had better achievement scores than students who used the traditional approach. The same researchers also found that students with low visual-spatial ability scored better on Coordinate Geometry tasks with Geogebra than their counterparts who used the traditional approach.

V. Theoretical Framework

This study is guided by the model-centred learning theory as well as the TPACK and Mathematical Knowledge for Teaching (MKT) frameworks. The model-centred learning theory says that learners can construct their own mental and/or physical models to aid cognition and learning (Seel, 2003). The theory also supports technology-integrated instruction whereby meaningful learning can be accomplished through the use of appropriate technology and instructional practices. For instance, learners can engage in modelling and simulations using appropriate dynamic technological systems which emphasize use of concrete scenarios, complexity management, and high-order decision-making (Bu, Spector & Haciomeroglu, 2011). According to the TPACK framework (http://tpack.org), effective teachers need, among other things, technological and pedagogical content knowledge generally, while specifically the MKT framework (with its various subdivisions and which also includes knowledge of ICTs) (Hill, Blunk, Charalambos, Lewis, Phelps, Sleep & Ball, 2008) is for effective teaching of mathematics. It would appear that learners could perform better if the teacher possesses the appropriate knowledge for teaching and uses the learner-centred methodologies, approaches, materials and technologies efficiently and effectively.

VI. Materials and Methods

Research on classroom practice using video analysis tools has been done (Rich, 2007; Savola, 2008), most of such researches being qualitative. However, in this study researchers did not have the statistical software with which to analyse data. Even if they had accessed the tools, they had to spend much more time and resources learning how to use the software. Hence the data was analysed 'manually' (through observation, counting and transcribing). The first researcher installed the free Geogebra software on the second researcher's (teacher's) laptop and asked her to go and learn the tutorials and activities on her own. After a brief trial-and-error session with Geogebra, the teacher-cum-researcher prepared an Advanced Level lesson on Transformation of Trigonometrical Functions covering Stretch, Shear Enlargement, Reflection and Rotation of the sine and cosine functions. Using systematic sampling, five out of 35 Advanced Level learners belonging to the teacher's class were selected. Avolunteer research assistant then took a video of the teacher while she was using Geogebra to teach and illustrate, to the five leaners, the relationship between trigonometrical functions, their graphs and transformations. The research assistant also took a video of one of the learners illustrating and demonstrating, u-

-sing Geogebra, the transformations of the trigonometrical functions.

During the TIMSS 1995 Video Study (Stigler, Gonzales, Kawanaka, Knoll & Serrano, 1999), data analysis was centred on lesson structure, interaction, discourse, pedagogical techniques, and content among others. But in the TIMSS 1999 Video Study (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs, Chui, Wearne, Smith, Kersting, Manaster, Tseng, Etterbeek, Manaster, Gonzales, & Stigler, 2003), the analysis then was about lesson structure, the nature of the mathematics presented, and the ways in which mathematics was considered in the lessons. But to analyse data in this study, the researchers observed/watched the videos at least five times, looked for the number of times certain terms and concepts were mentioned or explained as well as the importance, usefulness or value attached to Geogebra. In particular, the researchers looked for:

1. Teachers' and learners' behaviours and reactions towards the videos and towards Geogebra - In particular their physical and facial expressions – were they happy, confident, confused, or shy? The use of language, vocabulary and discourse were also checked and analysed.
2. Teachers' and learners' content knowledge (knowledge of trigonometrical functions, their relationships and their transformations). Trigonometrical functions and Transformations are topics which should be covered in the new Zimbabwe Advanced Level Math curriculum. (Ministry of Primary and Secondary Education, 2015c).
3. Teachers' and learners' pedagogical knowledge - how to teach content to the learners using the technology, how one learner illustrated and explained to other learners using the technology.
4. Teachers' and learners' technological knowledge (use and knowledge of ICT, use and knowledge of Geogebra).

Then the researchers built and categorised themes using thematic analysis method suggested by Braun and Clarke (2006), as cited in Maguire and Delahunt (2017).

It was during the lockdown restrictions imposed as a result of COVID-19 that this research was conceived at the same time considering the advantages of using video based lessons and the Geogebra software.

VII. Results and Findings

Pre-Lesson Planning

The teacher, aged around 40-50 years, is qualified with a teaching certificate, a BEd (Sc) and MEd (Sc) degrees. She is currently studying for a PhD with one of the local universities. She teaches Ordinary Level and Advanced Level mathematics at a local high school. However, she claimed she had never taught math using videos before. She had also not heard of nor used Geogebra before, but she was willing and enthusiastic to learn. So the other researcher installed the free Geogebra software on her laptop and asked her to go and learn the tutorials and activities on her own. Her reaction was, "Wow, I am afraid I will take some bit of time to do trial and error, but I must endure and try to 'catch the tricks' for my own professional development and for imparting the skills to my learners."

After the researchers had brainstormed on what to do, the teacher then prepared an Advanced Level lesson on

Transformation of Trigonometrical Functions. In particular, she intended to cover Stretch, Shear, Enlargement, Reflection and Rotation of the sine and cosine functions, paying some specific attention to what happens to the period and amplitude of the function both along the x and y axes. The tangent function, which is a quotient of the sine and cosine functions, was however, deliberately left out. Probably the teacher had her own reasons for that. It was then agreed that the teacher would be captured on video while demonstrating and explaining some trigonometrical transformations to her learners and later on, one of the learners would also use the Geogebra software to explain and illustrate some transformations to the teacher as well as to his or her peers.

The Actual Lesson (Teacher's Demonstration)

The teacher explained quite well the purpose of the lesson. It seemed that her previous 'trials and errors' had made her aware that Geogebra could be used to show or explain transformations of trigonometrical functions by observing and interpreting the relationships between their algebraic and graphical representations. The concepts of scale factors and direction (x or y direction) of enlargement or stretch were clearly explained and illustrated. At the end, the teacher concluded by encouraging the learners to go and try functions such as quadratics, cubic and other polynomials. Thus, she alluded to the fact that Geogebra could be used to teach a variety of math topics. However, she did not elaborate on the possible teacher or learner related challenges of the software. This could be a topic for another research.

Learners' Participation

After the teacher's demonstration, it was time for learners' hands-on activities. Each of the five learners had a laptop and was asked to enter on the keyboard and describe the characteristics of the emerging graph as well as its relationships with the original one. Functions of the form y = cos(x), y = -cos (x), y = cos (-x), y = cos (2x), and y = 3cos (x) were given to the learners to sketch and describe. Three of the learners described the graphs and the transformations very clearly and accurately. Apart from the skills of entering functions on the key board, the following concepts were also mastered: reflection in the x axis (for y = cos(x) and y = -cos(x)), stretch along x axis, scale factor 3, x coordinates do not change, scale factor ½, stretch along x axis, x values are halved, y values not affected. One learner gave a video presentation.

Observed Teacher and Learner Challenges

The researcher-cum-lesson observer noted that the teacher did not clearly link, or explain, how what was taught and done could be applied (giving specific examples) to real life situations. In other words, the usefulness of what was learnt appeared to be more theoretical than practical. Some of the leaners' challenges were identified by the observer as being slow to type on the laptop. Also according to the observer, the teacher's modelling or simulation ideas and activities seemed to be at the rudimentary level and were not adequately articulated or understood. Thus, linking theory with practice seemed to be a challenge to the teacher under the circumstances.

It was also noted that during both presentations, the other leaners did not interrupt the teacher nor the leaner who gave a presentation. Thus, the learners were disciplined throughout the lesson.

After the Video Demos

After the video demonstrations, the learners participated (without being video-graphed) by practicing individ-

-ually on their laptops the exercises given by the teacher. The teacher moved around the class assisting the leaners and commenting on their work.

Building and Categorising Themes

The following steps adapted from Braun and Clarke (2006) as cited in Maguire and Delahunt (2017) were followed:

Step 1: Become familiar with the data - To become familiar with the data, the researchers independently watched the videos at least five times as per plan.

Step 2: Generate initial codes - The researchers used open coding, i.e., there were no pre-conceived codes.

Step 3: Search for themes - To search for and come up with the themes, the researchers looked at the research questions and the purpose of the study.

Step 4: Review themes - The researchers identified and reviewed the themes to come up with those listed in Step 5.

Step 5: Define themes- The researchers defined emerging nine themes as: (a) The purpose of Geogebra, (b) Teacher's content knowledge of trig functions and transformations, (c) Teacher's 'Geogebra-literacy,' (d) Learner's content knowledge of trig functions and transformations, (e) Learner's 'Geogebra-literacy,' (f) Modelling and Simulations with Geogebra, (g) Real-life applications of Geogebra, (h) Teacher's attitudes and behaviours, and (i) Learners' attitudes and behaviours.

Step 6: Write-up - The following themes, though not assumed to be exhaustive or mutually exclusive, finally emerged. For instance, Purpose of Geogebra and Geogebra-literacy could be viewed as one theme just as Modelling and Simulations with Geogebra and Real-life applications of Geogebra could be merged. However, Purpose of Geogebra was considered to be a theme to be unveiled by the teacher while Geogebra-literacy was considered as a teacher-related as well as leaner-related theme. Modelling and Simulations was considered a classroom activity while Real-life applications of Geogebra could be an 'out of classroom' activity. Hence, the initial themes were left as they were and the results are indicated in Table 1 below.

Table 1. Thematic analysis results.

Themes	**Examples of Codes**	**Frequency of Occurrence, Teacher and Leaner Characteristics and Importance/Value Attached to Theme**
1. Purpose of Geogebra	To draw graph of trig function, To solve algebra and geometry problems, To model real life situations.	Using Geogebra to draw graphs of trig functions was mentioned by teacher 2 times, No other purpose was mentioned.
2. Teacher's content knowledge of trig functions and transformations	Familiarity with equations y = sin x, y = cos x and their variants, Familiarity with concepts of shear, stretch, rotation, enlargement, reflection.	Teacher had good and convincing knowledge of trig functions y = sin x, y = 3sin x, amplitude, period, etc., and transformations like stretch, shear, enlargement, rotation, reflection, scale factor.
3. Teacher's 'Geogebra-literacy' (technological	Mentioning purpose/function of Geogebra, Graphing functions using Geogebra,	Teacher first used chalkboard to write trig functions and transformations that will be demonstrated using

Themes	Examples of Codes	Frequency of Occurrence, Teacher and Leaner Characteristics and Importance/Value Attached to Theme
content knowledge)	Giving examples of modelling/simulations, How teacher uses the software e.g., typing speed, correcting errors.	Geogebra. Now using Geogebra teacher had slow to moderate typing speed in contrast with the learner who was a bit faster, Teacher improved as the lesson progressed, Mentioned Geogebra 4 times in her presentation and once when commenting about learner's presentation, Mentioned Geogebra's main purpose as that of graphing functions.
4. Learner's content knowledge of trig functions and transformations	Familiarity with equations y = sin x, y = cos x and their variants, Familiarity with concepts of shear, stretch, rotation, enlargement, reflection.	Quite good. Had grasped concepts of amplitude, period and range of trig function, but confused y values with intercept, mixed radians with degrees, Teacher corrected the learner. Leaner however needed more practice.
5. Learner's 'Geogebra-literacy' (technological content knowledge)	Purpose/function of Geogebra, Graphing functions using Geogebra, Giving examples of modelling/simulations, How learner uses the software e.g., typing speed, overcoming errors.	Had good typing speed in contrast with the teacher's. Seemed to know how to use the software although did NOT mention the term Geogebra at all.
6. Modelling and simulations with Geogebra	Mentioning some applications or uses of Geogebra not necessarily related to trig functions or transformations, Applications of Geogebra outside the classroom – making real objects which are modelled by Geogebra.	Modelling not adequately articulated and not used to come up with interesting patterns apart from just varying the functions such as y = sin x to y = sin 2x, etc.
7. Real-life applications of Geogebra	Geogebra-created graphs to make e.g., water fountain, Dam site selection and measurements using Geogebra, Making artefacts e.g., clay pots or houses with Geogebra created structures, patterns and measurements.	Not adequately articulated by both teacher and leaners. The teacher only mentioned Geogebra once and the term 'application' twice but such application was related to sketching graphs and then to describing the transformations only.
8. Teacher's attitudes and behaviours towards the whole lesson	Perceptions: e.g., Geogebra is easy/difficult to use, It is time wasting etc.	Positive but at first hesitant, Teacher ended her presentation by encouraging learners and giving exercises/homework. In learner's presentation teacher motivated and thanked learner for a good presentation. However, teacher's facial and other physical expressions were not captured by videographer.
9. Learners' attitudes and behaviours towards the whole lesson	Perceptions: e.g., Geogebra is easy/difficult to use, It is time wasting etc.	Positive, seemed to be quite motivated, There were observable smiles on the leaners' faces.
10. Use of video for teaching and learning	Advantages of video lesson, Disadvantages of video lesson, Can we learn/teach Geogebra applications without video?	The term 'video' was NOT mentioned by either teacher or leaner. However, observer noted that the videographer took much time focussing on the screen than on the whole classroom environment. Ethical considerations were met since both teacher and leaner presenters voluntarily agreed to be captured and were not 'video-shy.' Leaner presenter seemed to be happy to be captured in the video whilst the videographer did not

Themes	Examples of Codes	Frequency of Occurrence, Teacher and Leaner Characteristics and Importance/Value Attached to Theme
		capture the teacher's physical characteristics, e.g. facial expressions, nodding, smiling, or frowning. The videographer did not capture the other learners who were present.

VIII. Discussion

Although, in this study, the video quality was not that very good, learners were happy and motivated to use the software and to be captured in the videos. This finding agrees with Choi (2010) who concluded that in his research students were motivated in learning mathematics with Geogebra. In this research it was noticed that indeed Geogebra was user friendly and this is in line with Mthethwa et al (2020, p.7) who say, "… the use of GeoGebra also encouraged learner engagement in, and communication regarding, their interactive experiences with the application." Thus, Geogebra-based video lessons can enhance the technological content knowledge of the learners. Also the teacher's beliefs about the use of Geogebra in the video lesson progressed positively. Some of the challenges or limitations to this research were that the sample size was rather small, the study was carried out during COVID-19 lockdown resulting in some eligible participants not taking part in the study, and that the researchers had no access to video analysis technology (Hiebert et al., 2003) but had to do it manually. With regard to the structure of the lesson, the situation in countries such as the USA is that a taught lesson is normally followed by practice while in Germany it is the reverse. However, in Zimbabwe video lessons at secondary school level are rare and thus there seems to be no agreed structure. Nevertheless, through the way the teacher proceeded with her lessons, it was noted that her content and pedagogical knowledge of transformations of trigonometrical functions via Geogebra- based video lesson was quite good and sufficient. It was noted that the leaners benefitted a lot from the teacher's as well as the other leaner's presentations.

IX. Conclusion

Geogebra is a free mathematical software that can be used to teach topics such as transformations of trigonometrical functions by simultaneously illustrating their algebraic/symbolic and graphical aspects. It can be used even when there is no internet connectivity. It motivates learners when they can actually do the hands-on activities and see for themselves the various emerging graphs and how they are related. Geogebra offers a vast range of opportunities such as mathematical modelling with various functions, shapes and colours. However, there are also some challenges such as new computer software anxiety, computer illiteracy and lack of mathematical understanding itself. Through the observations of the demonstrations and the results and findings, the research questions of this study were answered. This study was carried out with a small sample and during the COVID-19 lockdown, hence it is not advisable to generalise the findings to a larger population. However, other teachers can still make use of Geogebra-based video lessons (with appropriate adjustments and editions) together with findings of this study to enhance their mathematics instruction.

X. Recommendations

Based on the findings of this study, and on the fact that online and ICT- enhanced learning have recently become the norm, the following recommendations are made:

- The Ministry of Primary and Secondary Education should ensure that math teachers and learners are ICT-literate and should provide the necessary materials and equipment.
- Teachers should use appropriate software such as Geogebra to impart modelling and simulation skills to their learners.
- Leaners should use skills such as modelling and simulations with Geogebra in real life applications and innovations.

Further research on teaching mathematics using videos and Geogebra could be carried out in Zimbabwe or elsewhere and with larger samples.

REFERENCES

[1] Bu, L., Mumba, F., Henson, H. & Wright, M. (2013). GeoGebra in Professional Development: The Experience of Rural Inservice Elementary School (K-8) Teachers, *Mevlana International Journal of Education (MIJE), Special Issue Dynamic and Interactive Mathematics Learning Environments, 3*(3), pp 64-76. DOI: http://dx.doi.org/10.13054/mije.si.2013.07

[2] Bu, L., Spector, J. M. & Haciomeroglu, E. S. (2011). Toward Model-Centered Mathematics Learning and Instruction Using GeoGebra: A Theoretical Framework for Learning Mathematics with Understanding. In Lingguo Bu & Robert Schoen (Eds.): Model-Centered Learning, *Pathways to Mathematical Understanding using GeoGebra,* Sense Publishers, Rotterdam, The Netherlands.

[3] Choi, K.S. (2010). Motivating students in learning mathematics with Geogebra, *Annals. Computer Science Series 8*(2), pp 65-76.

[4] Department of Education (2003). *National Curriculum Statement Grades 10-12 (General) Mathematics*. Pretoria: Department of Education. Retrieved on 24/3/20 from https://www.schoolnet.org.za/teach10/resources/eo/resources/curriculum/fet/Maths.pdf

[5] Government of Zimbabwe (2016). *Zimbabwe National Policy for Information and Communications Technology (ICT)*. Retrieved on 3/12/20 from https://en.unesco.org/creativity/sites/creativity/files/qpr/zimbabwe_national_policy_for_ict_2016_-_final_1.pdf

[6] Heid, M. K. (2002). Computer Algebra Systems in Secondary Mathematics Classes: The Time to Act is Now! *Mathematics Teacher 'Soundoff", 95*(9), pp 662-667

[7] Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., Chui, A. M., Wearne, D., Smith, M., Kersting, N., Manaster, A., Tseng, E., Etterbeek, W., Manaster, C., Gonzales, P., & Stigler, J. (2003). *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*, NCES (2003-013), U.S. Department of Education. Washington, DC: National Center for Education Statistics. Available online at http://timssvideo.com/sites/default/files/TIMSS%201999%20Math%20Report.pdf

[8] Hill, H. C., Blunk, M. L, Charalambos, C. Y., Lewis, J. M., Phelps, G. C, Sleep, L. & Ball, D. L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study, *Cognition and Instruction, 26*(4), pp 430-511, DOI: 10.1080/07370000802177235

[9] Hohenwarter, M., Hohenwarter, J., Kreis, Y. & Lavicza, Z. (2008). Teaching and learning calculus with free dynamic mathematics software GeoGebra. 11th International Congress on Mathematics Education, *ICME Mexico*, pp 1-9 Retrieved on 3/12/20 from https://www.researchgate.net/publication/228869636

[10] Hohenwarter, M., Jarvis, D. & Lavicza, Z. (2009). Linking Geometry, Algebra, and Mathematics Teachers: *GeoGebra* Software and the Establishment of the International GeoGebra Institute. *International Journal for Technology in Mathematics Education, 16*(2), pp 84-86.

[11] Kondratieva, M. (2013). Geometrical Constructions in Dynamic and Interactive Mathematics Learning Environment. In *Mevlana International Journal of Education (MIJE)), Special Issue: Dynamic and Interactive Mathematics Learning Environment 3*(3), pp 50-63, DOI: http://dx.doi.org/10.13054/mije.si.2013.06

[12] Lavicza, Z. & Koren, B. (2015). New Challenges in Developing DynamicSoftware for Teaching Mathematics. In S.J. Cho (ed.), *The Proceedings of the 12th International Congress on Mathematical Education*, DOI 10.1007/978-3-319-12688-3_72

[13] Ljajko, E. & Ibro, V. (2013). Development of ideas in a GeoGebra – aided mathematics instruction, *Mevlana International Journal of Education (MIJE), Special Issue: Dynamic and Interactive Mathematics Learning Environments, 3*(3), pp. 1- 7.

[14] Maguire, M. & Delahunt, B. (2017). Doing a Thematic Analysis: A Practical, Step-by-Step Guide for Learning and Teaching Scholars, *AISHE-J 9*(3), pp 3351-33514.

[15] Marinoni, G. van't Land, H. & Jensen, T. (2020). *The Impact of COVID-19 on Higher Education around the World: IAU Global Survey Report*, International Association of Universities, UNESCO House, Paris

[16] Mauchi, F.N., Karambakuwa, R.T., Gopo, R.N., Kosmas, N., Mangwende, S. & Gombarume, F.B. (2011). Entrepreneurship education lessons: A case of Zimbabwean tertiary education institutions. *Educational Research 2*(7), pp. 1306-1311.

[17] Ministry of Higher and tertiary Education, Science and Technology Development (n.d.). *Education 5.0: Doctrine for the Modernisation and industrialisation of Zimbabwe through Education, Science and Technology Development to Achieve Vision 2030*, Ministry of Higher and tertiary Education, Science and Technology Development, Zimbabwe. Retrieved on 9/5/2020 from https://library.buse.ac.zw/docs/gvt-publications/higher-edu-plan.pdf

[18] Ministry of Primary and Secondary Education (2017). *Secretary's circular No 2 of 2017: Implementation of the curriculum framework 2015-2022*. Harare, Zimbabwe

[19] Ministry of primary and secondary education (2015a). *Additional Mathematics Syllabus Forms 5 – 6: 2015-2022*. Curriculum Development and Technical Services, Harare, Zimbabwe

[20] Ministry of primary and secondary education (2015b). *Pure Mathematics Syllabus Forms 3 – 4: 2015-2022*. Curriculum Development and Technical Services, Harare, Zimbabwe

[21] Ministry of primary and secondary education (2015c). *Pure Mathematics Syllabus Forms 5 – 6: 2015-2022*. Curriculum Development and Technical Services, Harare, Zimbabwe

[22] Mthethwa, M., Bayaga, A., Bosse, M. J. & Williams, D. (2020). GeoGebra for learning and teaching: A parallel investigation, *South African Journal of Education, 40*(2), pp 1-12.

[23] NCTM (2000). *Principles and standards for school mathematics*. NCTM, Reston, VA.

[24] Pierce, R. & Stacey, K. (2011). Using Dynamic Geometry to Bring the Real World into the Classroom. In Lingguo Bu & Robert Schoen (Eds.): Model-Centered Learning, *Pathways to Mathematical Understanding Using GeoGebra,* Sense Publishers, Rotterdam, The Netherlands
[25] Rich. P.J. (2007). *The video analysis tool: Analyzing, assessing and adapting preservice teachers' instructional decisions* [Unpublished Doctor of Philosophy Thesis submitted to The University of Georgia, Athens]
[26] Saha, R. A., Ayub, A.F.M., & Tarmizi, R. A. (2010). The effects of GeoGebra on Mathematics Achievement Enlightening Coordinate Geometry Learning, *Procedia Social and Behavioral Sciences 8,* pp 686–693.
[27] Savola, L.T. (2008). *Video-based analysis of mathematics classroom practice: Examples from Finland and Iceland*. [Unpublished Doctor of Philosophy Thesis submitted to The Graduate School of Arts and Sciences, Columbia University].
[28] Seel, N. M. (2003). Model-centered learning and instruction. *Technology, Instruction, Cognition and Learning, 1*(1), pp 59–85.
[29] Sofoluwe, A.C., Shokunbi, M.O., Raimi, L. & Ajewole, T. (2013). Entrepreneurship Education as a Strategy for Boasting Human Capital Development and Employability in Nigeria: Issues, Prospects, Challenges and Solutions. *Journal of Business Administration an Education, 3*(1), pp. 25-50.
[30] Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S. & Serrano, A. (1999). *The TIMSS Videotape Classroom Study: Methods and Findings from an Exploratory Research Project on Eighth-Grade Mathematics Instruction in Germany, Japan, and the United States,* U.S Department of Education. National Center for Education Statistics, Washington, DC.
[31] TPACK framework. Available: http://tpack.orghttp://tpack.org

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